Exam Reasoning with Uncertainty (Dec. 2005)

1 Representing uncertainty

(1) Show that the following inequality holds for the inner measure:
$\mu_*(U \cup V) \geq \mu_*(U)+\mu_*(V)$ for disjoint U,V [superadditivity]. For the proof use the definition of inner measure.

2 Bayesian networks and rational decisions

(2) Two astronomers, in different parts of the world, make measurements M_1 and M_2 of the numbers of stars N in some small region of the sky, using their telescopes. Normally, there is a small possibility of error by up to one star (say 2%). Each telescope can also (with a slightly smaller probability, say 1%) be badly out of focus (events F_1 and F_2), in which case the scientist will **undercount** by one star (with a probability of 20%). Construct a belief network and give the (conditional) probability tables. For the number of stars, consider only the possible values 0,1,2 and assume the corresponding (a priory) probabilities 0.5, 0.3, 0.2.

(3) Assume that $\mu_X(U \cap V) = 0.2$, $\mu_X(U) = \mu_X(V) = 0.4$, and $\mu_X(U \cup V) = 0.7$.
a. Construct a Dutch book against X (and prove that your construction in fact is a Dutch book).
b. Why it's not possible to construct a Dutch book against X if $\mu_X(U \cap V) = 0.1$ (instead of 0.2)?

3 Dempster-Shaver Theory

(4) Suppose that a bag contains 10 marbles; 5 are known to be red, and the remainder are known to be blue or green, although the exact proportion of blue and green is not known.
a. First, define a mass function that describes this situation.
b. Second, calculate the value of the belief and plausibility function for the case that a marble taken out of the bag is (i) blue? (ii) red or green (iii) blue or green.
Hint: $W = \{R, B, G\}$

(5) Let be $W = \{1,2,3,4,5,6\}$. The mass functions over W are as follows: $m_1(\{1,2\}) = 1/6$, $m_1(\{3,4\}) = 1/3$, and $m_1(\{5,6\}) = \frac{1}{2}m_2(\{1\}) = 1/6$, $m_2(\{2\}) = 1/6$, $m_2(\{3\}) = 1/6$, $m_2\{4\}) = 1/6$, $m_2(\{5\}) = 1/6$, and $m_2(\{6\}) = 1/6$.

Calculate the mass function $m_1 \oplus m_2$, the belief function $Bel_1 \oplus Bel_2$, and the plausibility function $Pl_1 \oplus Pl_2$.

Fuzzy Logic

(6) Show that $f(x,y) = (xy)^n$ is a t-norm if and only if n=1 (n a natural number)

(7) Let A = [-1, +1] the crisp set consisting of all real numbers in the interval with the endpoints -1 and +1. Let R be the fuzzy relation $\mu_R(x, y) = e^{-(x-y)^2}$. Give an explicit expression for the relational composition of A with R

Fuzzy composition $\mu_{A \circ B}(y) = \sup_{x \in X}(\min(\mu_A(x), \mu_B(\langle x, y \rangle)))$

(8) Is the *modus ponens* valid in fuzzy logic? Prove it or give a counterexample!

Task Nr.	1	2	3	4	5	6	7	8	
Points	3	5	4	3	4	4	4	3	Max = 30