Probabilistic Robotics, BAIPR6, Fall 2009
Exam: Localization, Mapping & Exploration

Thursday December 17th, 13:00 - 16:00, room B4.24

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December 9, 2009

Question 1

This question focuses on the prediction step of an Extended Kalman Filter. A pose estimate at time step t is represented with a Gaussian with mean $\bar{\mu}_t$ and a covariance matrix $\bar{\Sigma}_t$. These are the corresponding update equations:

$$\bar{\mu}_t := g(\mu_{t-1}, u_t) \qquad \bar{\Sigma}_t := G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T$$

a) Give the state prediction function

Assuming an odometry model as introduced in section 5.4 of the book [1], the control vector u_t consist of three elements: an initial rotation angle δ_{rot_1}, a straight forward motion distance δ_{trans} and a final rotation angle d_{rot_2}. The state is a pose estimate, which is represented with a location x_t, y_t and a heading θ_t when we use the flat floor assumption.

Write out the function $g(\mu_{t-1}, u_t)$ of the odometry model. Note that equation 5.40 was corrected in the 3rd edition of the book [1].

b) Derive the Jacobians

To perform the linearization of the previous defined function g, derive the both Jabobians:

$$G_t = \frac{\partial g(\mu_{t-1}, u_t)}{\partial \mu_{t-1}} \qquad V_t = \frac{\partial g(\mu_{t-1}, u_t)}{\partial u_t}$$

c) Compute the estimates

M is the motion noise matrix in control space. With $V_t M_t V_t^T$ it is linearized and mapped into state space:

$$M = \begin{pmatrix} \sigma^2_{drot_1} & 0 & 0 \\ 0 & \sigma^2_{dtrans} & 0 \\ 0 & 0 & \sigma^2_{drot_2} \end{pmatrix} \qquad (1)$$

Assume $\sigma^2_{drot_1} = 5^o$, $\sigma^2_{dtrans} = 0.5m$ and $\sigma^2_{drot_2} = 10^o$. Given the initial estimates μ_0 and Σ_0 and the control vectors u_1 and u_2, compute the resulting estimates μ_1, Σ_1 and μ_2, Σ_2 using the Kalman Filter equations.

$$\mu_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \Sigma_0 = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix} \quad u_1 = \begin{pmatrix} 10^o \\ 3m \\ 10^o \end{pmatrix} \quad u_2 = \begin{pmatrix} -20^o \\ 10m \\ -20^o \end{pmatrix}$$

Question 2

A binary Bayes filter assumes that a cell is either occupied or unoccupied, based on the noisy evidence provided by a sensor for the correct hypothesis. In this question, you will be asked to build maximum likelyhood estimator for a cell, which provides the probability p that the next sensor reading indicates 'occupied'.

a) Compute a maximum likilyhood probability

The sensor is binary and can only measure "0=unoccupied" or "1=occupied". Suppose that the sensor receives a sequence

$$0, 0, 1, 0, 1, 1, 1, 0, 1, 0.$$

What is the maximum likelyhood probability p for the next reading to be 1?

b) Provide a general formula

Provide an incremental formula for a general maximum likelyhood estimator for this probability p.

c) Discuss the difference with a binary Bayes filter

Discuss the difference of this maximum likelyhood estimator to the binary Bayes filter (all for this single cell only).

Question 3

The full SLAM posterior can be written in the factored form:

$$p(x_{1:t}, m|z_{1:t}, u_{0:t-1})) = p(x_{1:t}|z_{1:t}, u_{0:t-1}) \prod_{n=1}^{N} p(m_n|x_{1:t}, z_{1:t}) \tag{2}$$

In the second factor of the factorization, the landmarks are supposed to be independent given the complete trajectory $x_{1:t}$ and the observations $z_{1:t}$. Is it possible to condition the map on the most recent pose x_t only? That is:

$$p(x_{1:t}, m|z_{1:t}, u_{0:t-1})) = p(x_{1:t}|z_{1:t}, u_{0:t-1}) \prod_{n=1}^{N} p(m_n|x_t, z_{1:t}) \tag{3}$$

Explain your answer.

Question 4

Consider a robot that operates in a triangular environment with three types of landmarks, as illustrated in Figure 1:

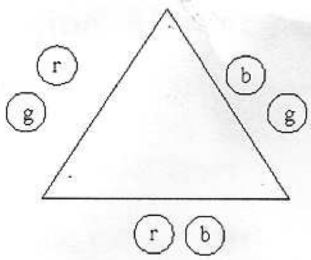

Figure 1: A triangular environment

Each arc is a location and each location has two different landmarks, each with a different color. Let us assume that in every round the robot can only inquire about the presence of one landmark type: either the one labeled "r", the one labeled "g" or the one labeled "b".

a) Clockwise

Suppose that robot first fires the detector for "b" landmarks and moves clockwise to the next arc. What would be the optimal landmark detector to use next?

b) Counterclockwise

How would the answer change if the robot moved counterclockwise to the next arc?

Success!

Acknowledgements

Two questions are based on assignments from the Albert-Ludwigs-Universität Freiburg, written by Wolfram Burgard.

References

[1] S. Thrun, W. Burgard and D. Fox, *Probabilistic Robotics (Intelligent Robotics and Autonomous Agents)*, The MIT Press, September 2005, ISBN 0-262-20162-3.

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